

## Minimal archi-texture for neutrino mass matrices

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**ABSTRACT:** The origin of the observed masses and mixing angles of quarks and leptons is one of imperative subjects in and beyond the standard model. Toward a deeper understanding of flavor structure, we investigate in this paper the minimality of fermion mass (Yukawa) matrices in unified theory. That is, the simplest matrix form is explored in light of the current experimental data for quarks and leptons, including the recent measurements of quark CP violation and neutrino oscillations. Two types of neutrino mass schemes are particularly analyzed; (i) Majorana masses of left-handed neutrinos with unspecified mechanism and (ii) Dirac and Majorana masses introducing three right-handed neutrinos. As a result, new classes of neutrino mass matrices are found to be consistent to the low-energy experimental data and high-energy unification hypothesis. For distinctive phenomenological implications of the minimal fermion mass textures, we discuss flavor-violating decay of charged leptons, the baryon asymmetry of the universe via thermal leptogenesis, neutrinoless double beta decay, and low-energy leptonic CP violation.

**KEYWORDS:** GUT, Supersymmetry Phenomenology, Neutrino Physics.

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## 1. Introduction

While the standard model successfully describes the gauge interactions of quarks and leptons, the origin of fermion masses and mixing angles is not yet understood. The standard model treats these quantities as free parameters, namely the coupling constants of Yukawa interactions. In the three-generation framework, Yukawa couplings are given by complex  $3 \times 3$  matrices in the generation space. A natural way to remedy the problem is to assume that some physics at high-energy scale dynamically controls Yukawa couplings and brings us the observed patterns of masses and mixing angles.

Along this line, various forms of Yukawa matrices have been discussed in the literature both for quarks and leptons. For the quark sector, possible small numbers of non-vanishing matrix elements have been examined [1] whether they are consistent with the experimental results, and the obtained forms of matrices have provided us clues to find symmetry or dynamics behind the Yukawa interactions. There have also been many studies about possible non-vanishing elements of lepton Yukawa matrices. In particular, neutrino mass matrices are classified from various standpoints [2] and that gives new insights about neu-

trino models to produce the observed data. Simultaneous treatments of quark and lepton Yukawa matrices lead to the investigations in unified theory based on the gauge groups such as  $SU(4)$  [3],  $SU(5)$  [4],  $SO(10)$  [5], and  $E_6$  [6]. An interesting point of unified theory is that the unification of standard model gauge groups implies non-trivial relations among various coupling constants including Yukawa couplings. The celebrated bottom/tau unification [7] and other theoretical predictions for masses and mixing angles [8] suggest that the unification hypothesis might provide us a deeper understanding of Yukawa matrices.

The recent experimental data on fermion masses and mixing angles, however, gives us some questions about the unification hypothesis. In particular, the neutrino oscillation experiments have been bringing out the structure of leptonic generation mixing which indicates some kind of quark-lepton asymmetry, that is, large lepton mixing against small quark mixing. In the light of quark-lepton unification, that seems somewhat unnatural since the generation structure of quarks is often correlated to that of leptons in unified theory. A possible solution to avoid such an intuitive result is to assume fermion mass matrices which are non-symmetric in the generation space [9]. This idea may have dynamical verification, e.g. in  $SU(5)$  unification where right-handed down-type quarks are combined with left-handed charged leptons. Consequently, since their mass matrices are generally left-right asymmetric, large leptonic mixing is derived without disturbing quark mixing angles, and can be consistent with unified gauge symmetry.<sup>1</sup> Given this fact, it is meaningful to explore general, not necessarily symmetric, forms of mass (Yukawa) matrices for quarks and leptons including the neutrino sector. With a throughout analysis in the light of the recent experimental data of quarks as well as leptons, new insights into the origin of generation structure are expected to be obtained.

In a previous work [11], we have analyzed in detail the quark sector where the down-quark mass matrices are of asymmetrical forms. Based on the minimality principle, namely with the smallest number of non-vanishing matrix elements, we have found 6 patterns of up and down-quark matrices which are entirely accommodated to the observed values of quark mass and mixing parameters. In addition, they are capable of leading to large leptonic mixing via  $SU(5)$  relation. In this paper, we complete the analysis by exploring lepton (neutrino) mass matrices with the minimal number of independent mass matrix elements.<sup>2</sup> We consider two types of schemes for neutrino masses. The first is to analyze light Majorana masses of the three-generation left-handed neutrinos, that is,  $3 \times 3$  Majorana mass matrices. The second scheme introduces three right-handed neutrinos which have large Majorana masses and also Dirac masses with the left-handed ones. In this scheme, we are lead to simultaneously analyze the minimality of Dirac and Majorana mass matrices. The purposes of this paper are to present the simplest forms of lepton mass matrices with which the 6 quark mass textures previously found are compatible and also to study some phenomenology related to the obtained forms of lepton mass matrices.

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<sup>1</sup>Leptonic mixing angles might be enhanced by integrating out heavy fields such as right-handed neutrinos while keeping correlative quark and lepton Yukawa couplings [10].

<sup>2</sup>The minimality principle for lepton mass matrices was also examined in [12]. Their analysis however focused on predictive neutrino mass matrices with specific types of charged lepton mixing, and has just a small overlap with ours. For example, the "Case C" in their classification is contained in our result.

The paper is organized as follows. In section 2, we describe the standard form of Yukawa interactions of quarks and leptons, and introduce several parameters and their experimental values needed in later analysis. In section 3, we revisit the quark mass textures found in the previous work to examine the discrepancy between down-type quarks and charged leptons, especially for the first and second generations. Based on these results, the minimality is explored for two types of neutrino mass models; Majorana mass matrices of left-handed neutrinos (in section 4) and Dirac and Majorana masses introducing right-handed neutrinos (in section 5). Section 6 is devoted to analyzing some phenomenological issues of the lepton mass textures found in the previous sections. Section 7 summarizes our results.

## 2. Formulation and input parameters

We first present the standard-model formulation of Yukawa interactions for quarks and leptons and the observed values of mass and mixing parameters, which are used as input parameters. The Yukawa terms invariant under the standard-model gauge transformations are given by

$$-\mathcal{L} = \overline{Q}_i(Y_u)_{ij}u_{Rj}H^* + \overline{Q}_i(Y_d)_{ij}d_{Rj}H + \overline{e}_{Ri}(Y_e)_{ij}L_jH^* + \text{h.c.}, \quad (2.1)$$

where  $Q$  and  $L$  denote the electroweak doublets of left-handed quarks and leptons,  $Q = (u_L, d_L)^T$  and  $L = (\nu_L, e_L)^T$ , and  $u_R, d_R, e_R$  are the right-handed up-type quarks, down-type quarks and charged leptons, respectively. The Yukawa coupling constants  $Y_u, Y_d$  and  $Y_e$  are complex-valued  $3 \times 3$  matrices ( $i$  and  $j$  are generation indices running over 1 to 3), and  $H$  is the Higgs doublet. There is no renormalizable Yukawa term for neutrinos in the standard model. After the electroweak symmetry breaking, these Yukawa interactions lead to the following quark and lepton mass terms:

$$-\mathcal{L} = \overline{u}_{Li}(M_u)_{ij}u_{Rj} + \overline{d}_{Li}(M_d)_{ij}d_{Rj} + \overline{e}_{Ri}(M_e)_{ij}e_{Lj} + \text{h.c.}, \quad (2.2)$$

$$(M_u)_{ij} = (Y_u)_{ij}v, \quad (M_d)_{ij} = (Y_d)_{ij}v, \quad (M_e)_{ij} = (Y_e)_{ij}v,$$

where  $v$  is the vacuum expectation value of the neutral component of Higgs scalar  $H$ . In addition to the above mass terms for charged fermions, one may consider Majorana mass term for left-handed neutrinos, taking into account the electroweak symmetry breaking. That is parametrized as

$$-\mathcal{L} = \frac{1}{2}\nu_{Li}^T(M_L)_{ij}C\nu_{Lj} + \text{h.c.}, \quad (2.3)$$

where  $C$  is the charge conjugation matrix. The Majorana mass matrix is generally complex-valued and has a symmetry property  $M_L^T = M_L$ . Throughout this paper, we assume that light neutrinos are Majorana particles.

The generation mixings are described by the Cabibbo-Kobayashi-Maskawa (CKM) matrix [13] for quarks and the Maki-Nakagawa-Sakata (MNS) matrix [14] for leptons, each of which consists of two unitary matrices

$$V_{\text{CKM}} = V_{uL}^\dagger V_{dL}, \quad V_{\text{MNS}} = V_{eL}^\dagger V_{\nu}. \quad (2.4)$$

These unitary matrices diagonalize the mass matrices of quarks and leptons as

$$M_u = V_{uL} M_u^D V_{uR}^\dagger, \quad M_d = V_{dL} M_d^D V_{dR}^\dagger, \quad (2.5)$$

$$M_e = V_{eR} M_e^D V_{eL}^\dagger, \quad M_L = V_\nu^* M_L^D V_\nu^\dagger, \quad (2.6)$$

where  $V_{uR}$ ,  $V_{dR}$  and  $V_{eR}$  are unitary matrices which can be removed by unitary rotations of right-handed fermions. The diagonal elements of  $M_u^D$ ,  $M_d^D$ , and  $M_e^D$  correspond to the experimentally observed mass eigenvalues;  $M_u^D = \text{diag}(m_u, m_c, m_t)$ ,  $M_d^D = \text{diag}(m_d, m_s, m_b)$ , and  $M_e^D = \text{diag}(m_e, m_\mu, m_\tau)$ , respectively, where all the eigenvalues are made real and positive. As for the neutrino masses,  $M_L^D = \text{diag}(m_1, m_2, m_3)$  and the squared mass differences are defined as  $\Delta m_{ij}^2 \equiv m_i^2 - m_j^2$  ( $i, j = 1, 2, 3$ ), which are the observables in neutrino oscillation experiments.

The current masses for quarks and charged leptons at the  $Z$ -boson mass scale are evaluated including various effects such as QCD coupling effects [15, 16]

$$\begin{aligned} m_u &= 0.000975 - 0.00260, & m_d &= 0.00260 - 0.00520, & m_e &= 0.00048685, \\ m_c &= 0.598 - 0.702, & m_s &= 0.0520 - 0.0845, & m_\mu &= 0.10275, \\ m_t &= 170 - 180, & m_b &= 2.83 - 3.04, & m_\tau &= 1.7467, \end{aligned} \quad (2.7)$$

in GeV unit. For the charged lepton masses, we have neglected errors and will refer to the above fixed values because the errors of charged lepton masses are very small and do not affect the analysis of mass matrix forms.

The physical mixing parameters in the quark sector are 3 mixing angles and 1 complex phase in the CKM matrix. The measured magnitudes of the CKM matrix elements are [15]

$$|V_{\text{CKM}}| = \left| \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \right| = \begin{pmatrix} 0.9739 - 0.9751 & 0.221 - 0.227 & 0.0029 - 0.0045 \\ 0.221 - 0.227 & 0.9730 - 0.9744 & 0.039 - 0.044 \\ 0.0048 - 0.014 & 0.037 - 0.043 & 0.9990 - 0.9992 \end{pmatrix}. \quad (2.8)$$

From various observations of CP-violating processes in the quark sector, the rephasing-invariant measure of CP violation [17] is given by

$$J_{\text{CP}} = (2.88 \pm 0.33) \times 10^{-5}. \quad (2.9)$$

In addition, the experimental study of the  $B$ -meson decay to charmoniums indicates [18]

$$\sin 2\phi_1/\beta = 0.726 \pm 0.037, \quad (2.10)$$

where  $\phi_1 = \beta$  is one of the angles of the unitary triangle for the  $B$ -meson system which is defined as  $\phi_1 = \beta \equiv \arg(V_{cd}^* V_{cb} V_{td} V_{tb}^*)$ .

The physical mixing parameters in the lepton sector are 3 mixing angles and 3 complex phases in the MNS matrix for Majorana neutrinos. We use the following parametrization for the mixing matrix:

$$V_{\text{MNS}} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix} \begin{pmatrix} e^{i\rho} \\ e^{i\sigma} \\ 1 \end{pmatrix}, \quad (2.11)$$

where  $c_{ij} \equiv \cos \theta_{ij}$ ,  $s_{ij} \equiv \sin \theta_{ij}$ , and  $\delta$  is the CP-violating Dirac phase. The parameters  $\rho$  and  $\sigma$  are the Majorana phases, which are irrelevant to neutrino flavor oscillations. In general, a  $3 \times 3$  unitary matrix contains 3 rotation angles and 6 complex phases, and a redefinition of lepton fields can remove 3 complex phases. Therefore the above parametrization covers the most general mixing matrix in the case that neutrinos are Majorana particles.

The information on the first and second generation neutrinos is extracted, e.g. from the solar neutrino experiments [19] and the KamLAND reactor anti-neutrino experiment [20]. The Super-Kamiokande experiment [21], the K2K long-baseline experiment [22] and the CHOOZ reactor experiment [23] probe the energy region  $\sim 10^{-(1-2)}$  eV and bring out the information including the third generation neutrino. Recent global analyses taking into account three-generation effects suggest [24]

$$\begin{aligned}
 \Delta m_{21}^2 &= 7.1 - 8.9 \times 10^{-5} \text{ eV}^2, \\
 |\Delta m_{31}^2| &= 1.4 - 3.3 \times 10^{-3} \text{ eV}^2, \\
 \sin^2 \theta_{12} &= 0.23 - 0.38, \\
 \sin^2 \theta_{23} &= 0.34 - 0.68, \\
 \sin^2 \theta_{13} &< 0.051,
 \end{aligned}
 \tag{2.12}$$

at the  $3\sigma$  level. One interesting observation is that the best fit value of the atmospheric angle  $\theta_{23}$  is the maximal one ( $45^\circ$ ), while the maximal mixture of the solar neutrinos is disfavored more than the  $5.6\sigma$  level [24]. As will be seen in later sections, the significant deviation of the solar angle  $\theta_{12}$  from the maximal one plays an important role to search for the minimal forms of neutrino mass textures.

### 3. Charged-fermion mass matrices

Before proceeding to the analysis of neutrino sector, we discuss the charged lepton mass matrix in unified theory based on the minimal quark mass matrices previously found. It turns out below that the minimal form of charged-fermion mass matrices is uniquely determined if one takes into account the discrepancy in mass eigenvalues between down-type quarks and charged leptons. In this section, after briefly reviewing the minimal forms of  $M_u$  and  $M_d$ , we examine a way to understand such discrepancy with group-theoretical factors in unified theory.

#### 3.1 The minimal mass matrices for quarks

In Ref. [11], we systematically investigated the minimality of quark mass matrices  $M_u$  and  $M_d$ , including the case that  $M_d$  is non-symmetric in the generation space. Here the term “minimality” means that the mass matrices successfully describe the experimental data with as few numbers of non-vanishing elements as possible. In other words, we have searched for the maximal number of vanishing matrix elements such that they are consistent with the observations. It is noted that the number of zero elements is that of independently-vanishing elements in each matrix. For example, for a symmetric matrix,

Solution	$M_u$	$M_d$
1	$\begin{pmatrix} a & & \\ & b & \\ & & b c \end{pmatrix}$	$\begin{pmatrix} d & e & \\ & h & \\ & & g f \end{pmatrix}$
2	$\begin{pmatrix} a & & \\ a & b & \\ & & b c \end{pmatrix}$	$\begin{pmatrix} d & e & \\ & h & \\ & & g f \end{pmatrix}$
3	$\begin{pmatrix} a & & \\ & b & \\ & & b c \end{pmatrix}$	$\begin{pmatrix} e & & \\ d & h & \\ & & g f \end{pmatrix}$
4	$\begin{pmatrix} a & & \\ a & b & \\ & & b c \end{pmatrix}$	$\begin{pmatrix} e & & \\ d & h & \\ & & g f \end{pmatrix}$
5	$\begin{pmatrix} a & & \\ & b & \\ & & b c \end{pmatrix}$	$\begin{pmatrix} e & & \\ & h & \\ d & g & f \end{pmatrix}$
6	$\begin{pmatrix} a & & \\ a & b & \\ & & b c \end{pmatrix}$	$\begin{pmatrix} e & & \\ & h & \\ d & g & f \end{pmatrix}$

**Table 1:** The 6 patterns of successful minimal mass textures.

“1 zero” implies that a diagonal element or a pair of off-diagonal elements in symmetric positions takes a negligibly small value compared to the others.

Through the exhaustive analysis based on such minimality principle, we have found that the 6 patterns of up and down quark matrices explain all the observations in the quark sector. Table 1 is the list of these 6 texture patterns. Each set of the minimal mass textures has 7 (= 3+4) zero elements (i.e. 8 (= 3+5) independent elements) in the up and down quark mass matrices, respectively. Furthermore it may be favorable that they are capable of leading to large leptonic 2-3 mixing (the atmospheric angle) via  $SU(5)$  relation. We have also found that there exist no solutions which explain large leptonic 1-2 mixing (the solar angle) at this level of minimality. Therefore the 6 patterns in table 1 should be fitted to the experimental results for leptons by being combined with a relevant form of neutrino mass matrices. When a large atmospheric angle originates from the charged lepton sector, a more economical form of neutrino mass matrices is expected to be found compared to the analysis where the charged lepton mass matrix is flavor diagonal.

It is relevant here to comment on the existence of physically equivalent mass textures. They are obtained by permuting rows and/or columns of the mass matrices in table 1, i.e. by exchanging generation indices of  $u_L$ ,  $u_R$ ,  $d_L$  and/or  $d_R$ . There are two types of operations which change the positions of zero matrix elements while preserving physical consequences for mass eigenvalues and mixing angles: (i) the exchanges of generation indices of  $d_R$ , and (ii) the exchanges of the same generation indices of  $u_L$  and  $d_L$  (and also  $u_R$  for a

symmetric  $M_u$ ). Mass textures obtained by these operations are also viable in the sense that they successfully describe the quark-sector observables. Note that, if the quark mass matrices are embedded into unified theory, a permutation of columns of  $M_d$  (i.e. of  $d_R$ ) might affect physical quantity in the lepton sector since a right-handed down-type quark and a left-handed charged lepton are often unified in a single representation of unified gauge group. However in the analysis including neutrino mass matrix, a simultaneous exchange of generation indices of  $d_R$ ,  $e_L$ , and  $\nu_L$  preserves physical predictions also in the lepton sector. Thus we conclude that it is sufficient to consider the 6 patterns presented in table 1 and to examine all flavor patterns of neutrino mass matrices, some of which are related to each other by permutations of generation indices.

In addition to the above two operations (i) and (ii), there is an approximate 2-3 exchange symmetry in  $M_d$ . In the parameter region where  $M_u$  and  $M_d$  properly reproduce the data of the quark sector, all  $M_d$  in table 1 lead to large 2-3 mixing of  $d_R$  so that  $V_{dR}$  becomes approximately invariant under the exchange of second and third rows. As will be seen, this feature would lead to some counterintuitive consequences in the analysis of neutrino mass matrices. We will mention this point in later discussion.

We have also found in the previous work the other possibilities of quark mass textures which have the same number of vanishing matrix elements (7 zeros) as the solutions in table 1:

$$M_u = \begin{pmatrix} a & & \\ a & d & b \\ & b & c \end{pmatrix}, \quad \begin{pmatrix} & a & \\ & b & d \\ a & d & c \end{pmatrix} \quad (3.1)$$

$$M_d = \begin{pmatrix} e & h & \\ & f & \\ & & g \end{pmatrix}, \quad \begin{pmatrix} e & & \\ & h & f \\ & & g \end{pmatrix} \quad (3.2)$$

All 4 ( $= 2 \times 2$ ) combinations of  $M_u$  and  $M_d$  well describe the present experimental data of the quark sector. An important difference between these 4 patterns and those in table 1 is the structure of down-quark mass matrix: the matrices in (3.2) do not have the 2-3 generation mixture and thus no significant contribution to the atmospheric angle arises when the matrices are embedded into  $SU(5)$  framework. We checked that this fact actually needs non-minimal, complex forms of neutrino mass matrices. In this paper, therefore, we concentrate on the mass textures shown in table 1 in the viewpoint of minimality.

### 3.2 The minimal mass matrix for charged leptons

In unified theory, it may be natural to assume matter unification which often leads to the same mass eigenvalues for down-type quarks and charged leptons. However when the observed values of these masses are extrapolated up to high-energy scale by renormalization group evolution, the following relations are found to be satisfied at the unification scale:

$$m_d \sim 3m_e, \quad 3m_s \sim m_\mu, \quad m_b \sim m_\tau. \quad (3.3)$$

To reproduce these mass relations, especially for the first and second generations, it is needed to introduce some unified symmetry breaking which splits the properties of quarks



and leptons. A typical example of such breaking effects is provided by group-theoretical factors [25], which originate in Yukawa interactions involving additional higher-dimensional Higgs fields. In this subsection, we examine the embedding of the minimal quark mass textures into unified theory, taking account of the effect of symmetry-breaking factors. There might be other possibilities for realizing (3.3) such as contributions from higher-dimensional operators, which bring the Yukawa sector additional free parameters to be tuned. These operators are also suppressed by the ratio between the unification scale and the gravitational scale. In connection with the minimality principle, we focus on the effect of group-theoretical factors which do not necessarily require additional parameters describing masses and mixing angles.

Let us first closely examine the minimal down-quark mass matrix  $M_d$ . There are 3 types of  $M_d$  in table 1:  $M_d^{(1)}$  for Solutions 1,2,  $M_d^{(2)}$  for Solutions 3,4, and  $M_d^{(3)}$  for Solutions 5,6. The mass hierarchy and mixing angles are properly reproduced with the following parametrizations:

$$\frac{M_d^{(1)}}{m_b} = \begin{pmatrix} \bar{d}\lambda^4 & \bar{e}\lambda^3 & \\ & \bar{h}\lambda^2 & \\ & \bar{g} & \bar{f} \end{pmatrix}, \tag{3.4}$$

$$\frac{M_d^{(2)}}{m_b} = \begin{pmatrix} & \bar{e}\lambda^3 & \\ \bar{d}\lambda^3 & & \bar{h}\lambda^2 \\ & \bar{g} & \bar{f} \end{pmatrix}, \tag{3.5}$$

$$\frac{M_d^{(3)}}{m_b} = \begin{pmatrix} & \bar{e}\lambda^3 & \\ & & \bar{h}\lambda^2 \\ \bar{d}\lambda & \bar{g} & \bar{f} \end{pmatrix}, \tag{3.6}$$

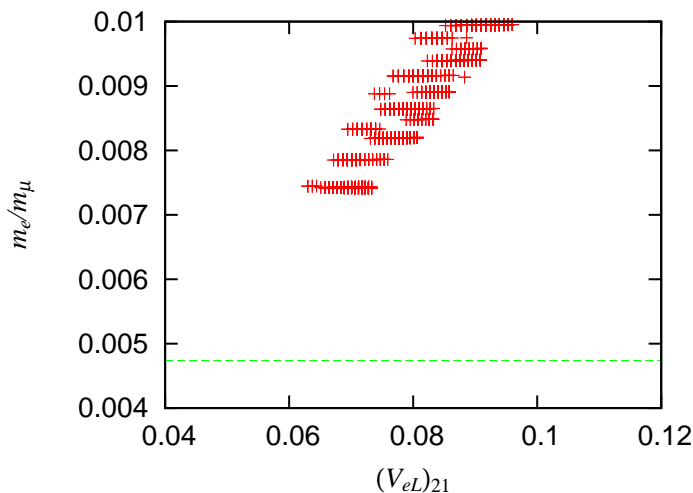
where  $\lambda$  is a small parameter of the order of the Cabibbo angle ( $\lambda \sim 0.2$ ), and  $\bar{d}$ ,  $\bar{e}$ ,  $\bar{f}$ ,  $\bar{g}$ ,  $\bar{h}$  are dimensionless  $\mathcal{O}(1)$  coefficients. It is found in Ref. [11] that the quark masses and mixing angles can be fitted by adjusting these coefficients. For any of the above forms of  $M_d$ , the mass eigenvalues for the second and third generations satisfy

$$\frac{m_s}{m_b} \simeq \bar{g}\bar{h}\lambda^2 \tag{3.7}$$

at the leading order of the expansion parameter  $\lambda$ . On the other hand, the mass eigenvalue of the first generation has rather different expressions:

$$\frac{m_d}{m_b} \simeq \begin{cases} \bar{d}\lambda^4 & \text{for } M_d^{(1)} \\ \frac{\bar{d}\bar{e}\bar{f}}{\bar{g}\bar{h}}\lambda^4 & \text{for } M_d^{(2)} \\ \frac{\bar{d}\bar{e}}{\bar{g}}\lambda^4 & \text{for } M_d^{(3)} \end{cases} \tag{3.8}$$

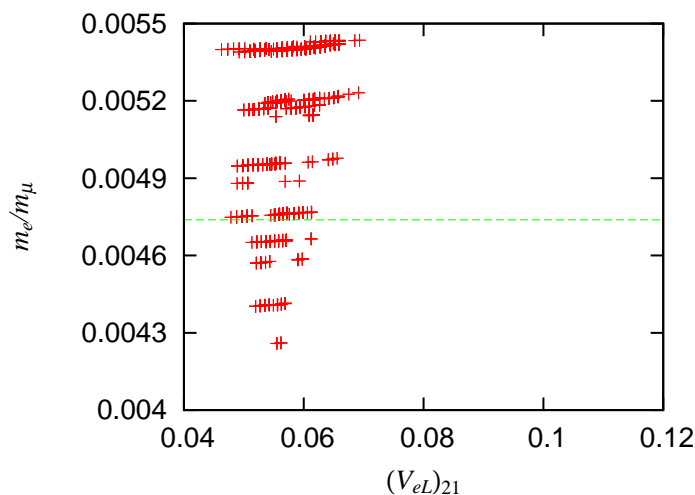
In unified theory framework, the charged lepton mass matrix  $M_e$  naively has the same form as  $M_d$ , which does not lead to the correct mass parameters in low-energy regime. To resolve it, we introduce the group-theoretical factor [25] assuming Higgs fields in higher-dimensional representations, e.g. 45 plet. of  $SU(5)$ . That induces relative factors “−3” in  $M_e$  compared to the corresponding matrix elements in  $M_d$ .



**Figure 1:** The predictions of the mass ratio  $m_e/m_\mu$  and mixing matrix element  $(V_{eL})_{21}$  for Solution 3. The horizontal dashed line shows the experimental value of the mass ratio.

It is first found from (3.3) and (3.7) that the element  $\bar{h}$  should be replaced with  $-3\bar{h}$  in  $M_e$  to bridge the gap between the strange quark and muon masses. Notice that the alternative choice, i.e. the factor  $-3$  for  $\bar{g}$ , may disturb the prediction  $m_b \simeq m_\tau$ , and we do not consider such a possibility in this paper. As for the first-generation mass eigenvalues, the relation (3.3) tells us that the electron mass is suppressed by the factor 3 compared to the down-quark mass. It is seen from (3.8) that this suppression can be realized only for  $M_d^{(2)}$  with the replacement  $\bar{h} \rightarrow -3\bar{h}$ . The replacement  $\bar{g} \rightarrow -3\bar{g}$  for  $M_d^{(2),(3)}$  also gives the same suppression effect for the electron mass, but in this case, the bottom-tau unification does not follow. We thus find that to introduce a group-theoretical factor  $-3$  for the element  $\bar{h}$  in Solutions 3,4 is the unique choice to properly reproduce the observed values of charged lepton masses at low energy. It may be interesting to notice that the resolution is economical in that a single attachment factor is sufficient to accommodate mass eigenvalues to the experimental relations (3.3) and no other modification is allowed for the minimal mass matrices.

We have shown that only Solutions 3 and 4 embedded in unified theory are consistent with the observed values of down-type quark and charged lepton masses. Let us see whether these solutions have parameter space where all other experimental constraints including the flavor mixing angles are satisfied. In figures 1 and 2, we show the parameter regions in which the quark mass data (2.7) and the experimental results (2.8)–(2.10) are explained. The horizontal axis shows the 1-2 generation mixture of left-handed charged leptons and the vertical one the mass ratio  $m_e/m_\mu$ . The horizontal dashed lines denote the observed value of  $m_e/m_\mu$  at the electroweak scale. It is found from these figures that Solution 4 accounts for the observed data in the quark sector as well as the charged lepton mass relations, but Solution 3 does not. We numerically checked that Solution 4 does explain all the quark-sector constraints listed in section 2 and also charged lepton mass eigenvalues.



**Figure 2:** The same as figure 1 for Solution 4.

There are two comments on the above analysis. First, figures 1 and 2 are drawn by using the experimental data at the electroweak scale. Since we are working in unified theory framework, the mass matrices are supposed to be given at the unification scale and should be compared to the data at that scale. However the conclusion obtained from figures 1 and 2 are stable against renormalization group effects. This is because radiative corrections to  $(V_{eL})_{21}$  and  $m_e/m_\mu$  do not involve any large coupling constants, and these quantities are almost energy-scale independent. The second is the prediction for the mixing matrix element  $(V_{eL})_{21}$ , i.e. the 1-2 generation mixing of left-handed charged leptons. It is found from figure 2 that, for the successful Solution 4, it takes the value  $\sim 0.07$  at most, which is not so large to solely provide the observed value of the solar angle. This fact will play an important role in classifying neutrino mass matrices in later sections.

In summary, we have found that only Solution 4 in table 1 can be accommodated to the charged lepton masses without contradicting the observations in the quark sector (2.7)–(2.10), if a group-theoretical factor is properly applied. The minimal mass matrices for charged fermions are thus given by

$$M_u = \begin{pmatrix} a & & \\ a & b & \\ & b & c \end{pmatrix}, \quad M_d = \begin{pmatrix} e & & \\ d & h & \\ & g & f \end{pmatrix}, \quad M_e = \begin{pmatrix} e & & \\ d & -3h & \\ & g & f \end{pmatrix}. \quad (3.9)$$

An interesting prediction of this combination of  $M_u$ ,  $M_d$  and  $M_e$  is a large 2-3 mixing angle in the charged lepton mixing matrix  $V_{eL}$ . In the parameter region where the experimental data of the quark sector is properly reproduced,  $V_{eL}$  is approximately invariant under the exchange of the second and third rows. Therefore a neutrino mass texture which is obtained by exchanging the generation labels of  $L_2$  and  $L_3$  predicts almost the same physics as the original one.

## 4. Majorana mass matrices for left-handed neutrinos

In this section, we study the matrix form of left-handed neutrino Majorana masses. The Majorana mass matrix  $M_L$  defined in (2.3) are derived in various ways, for example, from the higher-dimensional operator  $HHL_i^T CL_j$ , the expectation value of electroweak triplet scalar field, etc. Keeping in mind such dynamical origins, we explore the minimality of effective mass matrix  $M_L$ , combining it with the successful minimal textures of charged fermions (3.9). Here we use the minimality in the same meaning as in section 3, that is, the maximal number of zero matrix elements which is consistent to all the experimental results for quarks and leptons.

### 4.1 The would-be minimal forms

Let us first examine the minimality of  $M_L$  by taking account of the information about neutrino mass eigenvalues only. Since only two mass-squared differences are measured in neutrino oscillation experiments, one of three-generation neutrinos can be exactly massless. Thus we expect that the minimal matrix  $M_L$  would have one zero and two nonzero mass eigenvalues to reproduce the data (2.12). This fact is a crucial difference from the texture analysis of quark mass matrices. Next, consider the neutrino generation mixing described by the matrix  $V_\nu$  which diagonalizes  $M_L$ . As for the generation mixing, the solution (3.9) provides a large 2-3 mixing for the lepton sector. Furthermore we have found that the 1-2 mixing of charged leptons is not so large enough to explain the observed solar neutrino oscillation. Therefore the neutrino mixing matrix  $V_\nu$  must involve a sizable mixture between the first and second generations.

The above discussion about neutrino masses and mixing restricts the candidates for the minimal matrix to the following forms:

$$M_L = \begin{pmatrix} l & \\ l & m \end{pmatrix}, \quad \begin{pmatrix} m & l \\ l & \end{pmatrix}. \quad (4.1)$$

Note that these two types of matrices give different physical predictions since the basis of left-handed leptons is already fixed by  $M_e$ . In addition to these matrices, there are two additional candidates for the minimal mass texture which are obtained by exchanging the second and third generation indices in (4.1). At first glance, it seems that those additional candidates provide only 1-3 generation mixing and do not work for viable solutions. However they should be included in the list since, as we mentioned before, the charged lepton mixing matrix  $V_{eL}$  for Solution 4 is approximately invariant under the permutation of the second and third rows. Therefore a neutrino mass matrix  $M_L$  always has such a doubling freedom while keeping physical consequences. In the following, we do not explicitly write down doubled candidates, though they are included in our analysis.

The matrices (4.1) seem to satisfy the criterions for neutrino masses and mixing, but it turns out that they are impractical if combined with the minimal solution (3.9). For example, let us see the first mass matrix in (4.1). (For the second matrix, a similar discussion can also be applied.) It is first noticed that the matrix predicts the spectrum that the third-generation neutrino is massless and the other two neutrinos should have

almost degenerate masses. That is,

$$\frac{m_1}{m_2} \simeq 1 - \frac{1}{2} \frac{\Delta m_{21}^2}{\Delta m_{23}^2}, \quad m_3 = 0. \quad (4.2)$$

A non-vanishing off-diagonal element  $(V_\nu)_{12}$  is expressed in terms of the mass eigenvalues:

$$(V_\nu)_{12} = \sqrt{\frac{m_1}{m_1 + m_2}}. \quad (4.3)$$

Thus the 1-2 mixing angle in  $V_{\text{MNS}}$  is given by

$$\sin \theta_{12} \simeq \frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}} (V_{eL})_{21}, \quad (4.4)$$

up to tiny corrections of order  $\mathcal{O}(10^{-3})$ . We have assumed  $(V_{eL})_{11} \simeq 1$  and taken phase degrees of freedom in Yukawa couplings so that the last term in (4.4) is destructive to account for  $\theta_{12} < 45^\circ$ . The experimental value of  $\theta_{12}$  (2.12) gives a bound on the magnitude of charged lepton mixing:

$$0.13 < (V_{eL})_{21} < 0.32 \quad (4.5)$$

at the  $3\sigma$  level. Therefore a crucial point required for the minimal mass matrices is whether a sizable contribution to  $(V_{eL})_{21}$  arises or not. We find from (3.9) an approximate analytic expression for charged lepton mixing:

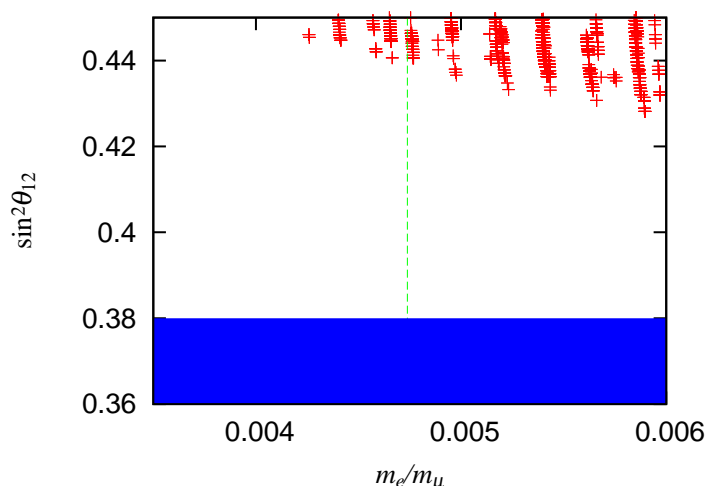
$$(V_{eL})_{21} < \frac{1}{3} \left( V_{us} + \sqrt{\frac{m_u}{m_c}} \right) \sim 0.09, \quad (4.6)$$

which is not consistent to the bound (4.5) (and see also the numerical result, figure 2). The inequality (4.6) implies that the group-theoretical factor in  $M_e$  prevents the mixing  $(V_{eL})_{21}$  from being large enough to fulfill the required value (4.5). In figure 3, we present a typical numerical result of the combination of (3.9) and the first matrix in (4.1). All the other experimental constraints are satisfied in this figure. It is found that the 1-2 mixing of charged leptons cannot be so large, which confirms the above analytic result. In the end, the left-handed Majorana mass matrix  $M_L$  (4.1) are experimentally disfavored at more than  $3\sigma$  level.

## 4.2 The minimal mass matrix for Majorana neutrinos

We have shown that two would-be minimal patterns of mass matrices (4.1) are not suitable to explain the solar neutrino oscillation. Let us consider the next level of minimality for the Majorana mass matrix  $M_L$  which has 3 vanishing matrix elements. There are 20 patterns of symmetric matrices with 3 zeros. By a detailed analysis of these 20 matrices, the following 8 patterns are found to be successful:

$$M_L = \begin{pmatrix} l & & \\ l & m & \\ & & n \end{pmatrix}, \begin{pmatrix} l & & \\ l & m & \\ & m & n \end{pmatrix}, \begin{pmatrix} l & m & \\ & m & n \\ & & \end{pmatrix}, \begin{pmatrix} l & & \\ & m & \\ l & m & n \end{pmatrix}, \quad (4.7)$$



**Figure 3:** A typical prediction of the matrix forms (3.9) and (4.1). The horizontal and vertical axes mean the mass ratio  $m_e/m_\mu$  and the solar angle, respectively. The filled (blue) region is experimentally allowed for  $\theta_{12}$  at the  $3\sigma$  level. The vertical dashed-line shows the observed value of the mass ratio.

Solution	$m_1$ (eV)	$m_2$ (eV)	$m_3$ (eV)	$\sin \theta_{13}$	$ \langle m_{ee} \rangle $ (eV)
$\begin{pmatrix} l \\ l \ m \\ n \end{pmatrix}$	0.0026 – 0.0075	0.0088 – 0.012	0.038 – 0.059	0.025 – 0.055	0.0003 – 0.0011
$\begin{pmatrix} l \\ l \ m \\ m \ n \end{pmatrix}$	0.0027 – 0.010	0.0089 – 0.014	0.040 – 0.060	0 – 0.15	0.0002 – 0.0014
$\begin{pmatrix} l \ m \\ m \ n \end{pmatrix}$	0.035 – 0.058	0.036 – 0.058	0	0.025 – 0.055	0.0084 – 0.058
$\begin{pmatrix} l \\ m \\ l \ m \ n \end{pmatrix}$	0	0.0084 – 0.0094	0.037 – 0.059	0.13 – 0.37	0.0002 – 0.0018

**Table 2:** The predictions of the minimal Majorana mass matrices  $M_L$  with the minimal solution (3.9). A matrix obtained by exchanging the second and third labels in each solution is also available which has almost the same physical predictions as the original one. In the first line,  $m_{1,2,3}$  are the mass eigenvalues, and  $\langle m_{ee} \rangle$  is the averaged mass parameter for neutrino-less double beta decay.

and four matrices obtained by exchanging the second and third-generation indices in (4.7). That is, the 8 matrices are classified into 4 types which are physically inequivalent. Any combination of (3.9) and one of these matrices is in accordance with the observed data of quarks and leptons (2.7)–(2.10) and (2.12). In the following, we discuss in order the phenomenological implications of the minimal forms of  $M_L$  (4.7). They have rather different predictions for physical quantities and can be distinguished. The predictions are summarized in the end of this section (see table 2).

1.

$$\begin{pmatrix} l & \\ l & m \\ & n \end{pmatrix} \quad \left[ \begin{pmatrix} l & \\ n & \\ l & m \end{pmatrix} \right] \quad (4.8)$$

The latter matrix (in the bracket) is obtained by exchanging the second and third labels of the former and has similar physical implications due to the approximate invariance of  $V_{eL}$  as mentioned before. It is first noticed that the matrix (4.8) is able to provide only 1-2 neutrino mixing. Thus the physical mixing angle  $\theta_{13}$  is given by

$$\sin \theta_{13} = |(V_{eL})_{31}| \simeq 0.025 - 0.055 \quad (4.9)$$

for the minimal solution (3.9). The numerical uncertainty mainly comes from those of the down and strange quark masses. As for the neutrino 1-2 mixing, we find an approximate relation:

$$\sin \theta_{12} \simeq |(V_{\nu})_{12}| = \sqrt{\frac{m_1}{m_1 + m_2}}, \quad (4.10)$$

where we have neglected  $\mathcal{O}(10^{-2})$  corrections from the charged lepton side.

In general, there are 3 types of neutrino mass spectrum allowed by the current experimental data: normal hierarchy ( $m_1 < m_2 \ll m_3$ ), inverted hierarchy ( $m_3 \ll m_1 < m_2$ ), and degenerate one ( $m_1 \simeq m_2 \simeq m_3$ ). For the mass matrix (4.8), the eigenvalues with normal hierarchy are expressed in terms of the observables in oscillation experiments:

$$\begin{aligned} m_1 &\simeq \sqrt{\frac{\Delta m_{21}^2 \tan^4 \theta_{12}}{1 - \tan^4 \theta_{12}}}, \\ m_2 &\simeq \sqrt{\frac{\Delta m_{21}^2}{1 - \tan^4 \theta_{12}}}, \\ m_3 &\simeq \sqrt{\Delta m_{32}^2 + \frac{\Delta m_{21}^2}{1 - \tan^4 \theta_{12}}}. \end{aligned} \quad (4.11)$$

On the other hand, it is easily found from (4.10) that the inverted hierarchy and degenerate spectrum lead to a nearly maximal mixing angle  $\sin \theta_{12} \sim 1/\sqrt{2}$ , and therefore cannot be predicted by the mass texture (4.8). Finally, the effective Majorana mass for neutrino-less double beta decay is given by  $|\langle m_{ee} \rangle| \simeq \sqrt{\Delta m_{21}^2 \sin 2\theta_{12} \tan 2\theta_{12}} |(V_{eL})_{21}|$ , and is found to be small.

2.

$$\begin{pmatrix} l & \\ l & m \\ m & n \end{pmatrix} \quad \left[ \begin{pmatrix} l & \\ n & m \\ l & m \end{pmatrix} \right] \quad (4.12)$$

The latter in the bracket is the label-exchanged matrix from the former. Diagonalizing the mass matrix, we find the off-diagonal elements  $(V_\nu)_{12}$  and  $(V_\nu)_{13}$  are given by

$$(V_\nu)_{12} = \sqrt{\frac{m_1(m_1^2 - m_3^2)}{(m_1 + m_2)(m_1^2 - m_1m_2 + m_2^2 - m_3^2)}}, \quad (4.13)$$

$$(V_\nu)_{13} = \sqrt{\frac{m_1m_2(m_1 - m_2)}{(m_1 - m_3)(m_2 + m_3)(m_1 - m_2 + m_3)}}. \quad (4.14)$$

If the first-two generations are almost degenerate ( $m_1 \simeq m_2$ ), i.e. for the inverted hierarchy or degenerate spectrum, the above relation indicates  $(V_\nu)_{12} \simeq 1/\sqrt{2}$ . That is unfavorable for the solar neutrino data since the electron has only slight mixture to the other generations in the minimal solution (3.9). On the other hand, for the normal hierarchy ( $m_{1,2} \ll m_3$ ), the formula of  $(V_\nu)_{12}$  is reduced to the relation (4.10) at the leading order of  $m_{1,2}/m_3$ , and also the mass spectrum (4.11) is reproduced up to small corrections. In addition, the neutrino 1-3 mixing angle becomes at the leading order

$$(V_\nu)_{13} \simeq \frac{m_2}{m_3} \sqrt{\frac{m_1}{m_3}} \simeq \frac{\tan \theta_{12}}{(1 - \tan^4 \theta_{12})^{3/4}} \left( \frac{\Delta m_{21}^2}{\Delta m_{32}^2} \right)^{3/4} \simeq 0.04 - 0.14. \quad (4.15)$$

Thus  $\sin \theta_{13}$  is of this order when there is no accidental parameter cancellation between the charged lepton and neutrino sectors, and would be detectable in future neutrino experiments.

3.

$$\begin{pmatrix} l & m \\ m & n \end{pmatrix} \quad \left[ \begin{pmatrix} l & m \\ m & n \end{pmatrix} \right] \quad (4.16)$$

An important feature of this mass texture is a vanishing determinant, which predicts the inverted hierarchy with a massless third-generation neutrino. As for the mixing angles, no 1-3 neutrino mixing follows and the physical lepton 1-3 mixing is therefore given by

$$\sin \theta_{13} = |(V_{eL})_{31}| \simeq 0.025 - 0.055 \quad (4.17)$$

for the minimal solution (3.9). Since the matrix has 3 free parameters for the first-two generations, there are no predictive relations, in particular, for the lepton 1-2 mixing angle. It is also interesting to notice that the 1-1 element of the matrix is nonzero, which might give a sizable ratio of neutrino-less double beta decay ( $0\nu\beta\beta$ ). With a massless third-generation neutrino, the averaged mass parameter responsible for  $0\nu\beta\beta$  is evaluated as

$$\langle m_{ee} \rangle \simeq m_1 \cos^2 \theta_{12} e^{-2i\rho} + m_2 \sin^2 \theta_{12} e^{-2i\sigma}, \quad (4.18)$$

where  $\rho$  and  $\sigma$  are the Majorana phases of neutrinos in  $V_{\text{MNS}}$ , and we have used  $\sin \theta_{13} \ll 1$ . In the parameter region where the experimental data (2.12) is explained,



the averaged neutrino mass takes the value  $|\langle m_{ee} \rangle| \simeq 0.0084 - 0.058$  eV. The planned improvements in the sensitivity to such a small mass parameter are expected to reach  $|\langle m_{ee} \rangle| \sim \mathcal{O}(10^{-2})$  eV [26], and this type of minimal matrix will be testable in near future.

4.

$$\begin{pmatrix} & l & \\ & m & \\ l & m & n \end{pmatrix} \quad \left[ \begin{pmatrix} l & & \\ l & n & m \\ m & & \end{pmatrix} \right] \quad (4.19)$$

This matrix also has a vanishing determinant. We find that it predicts the spectrum with normal hierarchy and a massless first-generation neutrino. The inverted mass hierarchy is not allowed since  $(V_\nu)_{12} \simeq 1/\sqrt{2}$  in the parameter region where the CHOOZ bound in (2.12) is satisfied, which leads to a too large solar angle. Another point of the matrix (4.19) is that it induces a relatively large value of lepton 1-3 mixing angle. For the normal hierarchy spectrum, one obtains a relation in the neutrino sector:

$$(V_\nu)_{13} = (V_\nu)_{12} \sqrt{\frac{m_2}{m_3}}, \quad (4.20)$$

which gives the following prediction among the observables:

$$\sin^2 \theta_{13} \simeq \sin^2 \theta_{12} \sqrt{\frac{\Delta m_{21}^2}{\Delta m_{32}^2}}, \quad (4.21)$$

up to small corrections from the charged lepton sector. Substituting the experimental data (2.12) for the right-handed side, we roughly obtain  $\sin \theta_{13} \simeq 0.18 - 0.32$ , which is marginal to the current experimental upper bound. The effective Majorana mass is predicted to be small:  $|\langle m_{ee} \rangle| \simeq 2(\Delta m_{21}^2 \Delta m_{32}^2)^{\frac{1}{4}} |(V_\nu)_{12} (V_{eL})_{31}| < 1.8 \times 10^{-3}$  eV, if taken account of the experimental bound on  $\theta_{13}$ .

## 5. The minimal mass matrices in the seesaw scenario

In the previous section, we have investigated possible forms of the Majorana mass matrix of left-handed neutrinos without referring to underlying mechanisms which realize tiny neutrino masses. One of the most attractive schemes for light neutrinos is the seesaw mechanism [27] to introduce heavy right-handed neutrinos and consider the following mass terms:

$$-\mathcal{L} = \overline{\nu_{Ri}} (Y_\nu)_{ij} L_j H + \frac{1}{2} \nu_{Ri}^T (M_R)_{ij} C \nu_{Rj} + \text{h.c.}, \quad (5.1)$$

where  $\nu_{Ri}$  denote right-handed neutrinos which are singlets under the standard model gauge group,  $Y_\nu$  is the neutrino Yukawa matrix, and  $M_R$  the Majorana mass matrix of right-handed neutrinos. If mass eigenvalues of  $M_R$  are sufficiently larger than the electroweak scale, one obtains effective Majorana mass matrix of left-handed neutrinos at low-energy regime:

$$M_L = -M_\nu^T M_R^{-1} M_\nu, \quad (5.2)$$

where  $M_\nu$  is the neutrino Dirac mass matrix:  $M_\nu = Y_\nu \langle H \rangle$ .

In this section, we examine the minimality of two mass matrices  $M_\nu$  and  $M_R$ , that is, the minimal (total) number of zero matrix elements which are compatible with the experimental data. Note that the minimality of  $M_\nu$  and  $M_R$  is different from that of  $M_L$ : as will be seen, the minimal forms of  $M_\nu$  and  $M_R$  do not necessarily lead to those of  $M_L$  found in the previous section. Therefore these two analyses of minimality should be independently performed. We assume that there exist no sterile states in the light spectrum. That implies the Majorana mass matrix  $M_R$  has a non-vanishing determinant, which is a contrast to the previous analysis where  $M_L$  may have a vanishing determinant. It is seen from the seesaw formula (5.2) that any exchange of the generation indices of right-handed neutrinos does not change physical consequences at low-energy regime. Further the label exchange of left-handed neutrinos  $\nu_{L2} \leftrightarrow \nu_{L3}$  also gives additional viable textures because of the approximate 2-3 exchanging symmetry of  $V_e$  as noted before. In the following, we do not particularly mention physically-equivalent label-exchanged matrices, though the results should always be interpreted as including such additional solutions.

It is first noticed that the minimal form of  $M_R$  with a non-vanishing determinant has 4 independent zeros. There are 3 types of matrix forms with such symmetric 4 zeros:

$$M_R = \begin{pmatrix} & r & \\ r & & \\ & & s \end{pmatrix}, \begin{pmatrix} & r & \\ & s & \\ r & & \end{pmatrix}, \begin{pmatrix} s & & \\ & r & \\ & & r \end{pmatrix}. \quad (5.3)$$

They are related to each other by permuting generation labels. The Dirac mass matrix  $M_\nu$  must have at least two nonzero eigenvalues to account for the neutrino oscillation data. Therefore  $M_\nu$  with 7 vanishing entries are the simplest candidates for minimality. There are 3 classes of such rank-two matrix  $M_\nu$  with 7 zeros:

$$M_\nu = \begin{pmatrix} p & \\ & q \end{pmatrix}, \begin{pmatrix} & p \\ p & \\ & q \end{pmatrix}, \begin{pmatrix} p & \\ & q \end{pmatrix}. \quad (5.4)$$

All the other matrices obtained by exchanging rows and/or columns need not to be included since the relabeling effect is already taken into account in  $M_R$  (5.3). It is easily found that, via the seesaw operation (5.2), any combination of 4-zero  $M_R$  (5.3) and 7-zero  $M_\nu$  (5.4) gives an effective Majorana mass matrix  $M_L$  with only one mass squared difference, and therefore is excluded by the oscillation experiment results.

The next simplest candidate is the combination of  $M_\nu$  and  $M_R$  with 10 vanishing matrix elements in total: 6-zero  $M_\nu$  + symmetric 4-zero  $M_R$  and 7-zero  $M_\nu$  + symmetric 3-zero  $M_R$ . We examined all matrix patterns whether they can totally fit the experimental data of quarks and leptons, and found that the following mass textures are almost successful:

- $M_\nu$  with 6 zeros +  $M_R$  with symmetric 4 zeros

$$M_\nu = \begin{pmatrix} p & & \\ & q & \\ & & r \end{pmatrix}, \begin{pmatrix} & p & \\ & q & \\ & & r \end{pmatrix}, \begin{pmatrix} p & q & \\ & q & \\ & & r \end{pmatrix}, \begin{pmatrix} p & q & \\ & q & \\ & & r \end{pmatrix}, \quad (5.5)$$

$$M_R = \begin{pmatrix} s & \\ & t \end{pmatrix}. \quad (5.6)$$

- $M_\nu$  with 7 zeros +  $M_R$  with symmetric 3 zeros

$$M_\nu = \begin{pmatrix} p & \\ & q \end{pmatrix}, \quad (5.7)$$

$$M_R = \begin{pmatrix} r & \\ r & s \\ & t \end{pmatrix}, \begin{pmatrix} r & \\ r & s \\ s & t \end{pmatrix}, \begin{pmatrix} r & s \\ s & \\ & t \end{pmatrix}, \begin{pmatrix} r & s \\ r & \\ s & t \end{pmatrix}. \quad (5.8)$$

In addition to these, the relabeling of generation indices of  $\nu_{R_i}$  in both  $M_\nu$  and  $M_R$  gives mass textures which have the same physical consequences. It is easily shown that all the above combinations lead to the Majorana mass matrix  $M_L$  of the form (4.1). Thus from the discussion in section 4.1, the solar angle  $\theta_{12}$  is found to be larger than the observed value, when the neutrino mass matrices are combined with the minimal mass matrices of charged fermions (3.9) in unified theory.

Consequently, we are lead to the minimal mass matrices of neutrinos in the seesaw scenario:  $M_\nu$  and  $M_R$ , which are compatible with the charged-fermion sector (3.9), contain 9 vanishing elements. There are 3 classes of such neutrino matrices:

- $M_\nu$  with 5 zeros +  $M_R$  with symmetric 4 zeros
- $M_\nu$  with 6 zeros +  $M_R$  with symmetric 3 zeros
- $M_\nu$  with 7 zeros +  $M_R$  with symmetric 2 zeros.

We exhausted all possibilities of these classes and found a variety of successful neutrino mass textures. In the case of 5-zero  $M_\nu$  and 4-zero  $M_R$ , 12 patterns of mass textures are consistent with all the observables for leptons as well as quarks (2.7)–(2.10) and (2.12) in collaboration with the minimal mass matrices (3.9). Note that the result includes 2 patterns of  $M_\nu$  and  $M_R$  which do not lead to the minimal form of  $M_L$  (4.7) but successfully describe all the observables. In the case of 6-zero  $M_\nu$  and 3-zero  $M_R$ , we found 25 available mass textures and, for 7-zero  $M_\nu$  and 2-zero  $M_R$ , only 6 patterns are viable. It may be interesting to note that each texture predicts a definite pattern of neutrino mass spectrum: there is no matrix form which explains the observed data both with the normal and inverted mass hierarchies.

The list of the minimal neutrino mass matrices in the seesaw scenario are summarized in the appendix. We also present for each solution the resultant Majorana mass matrix  $M_L$ , possible mass spectrum, and some phenomenological implications, which might play a key role for the matrices to be testified.

## 6. Phenomenological implications of minimal mass matrices

In the previous section, we have found considerable patterns of seesaw-type mass textures compatible with the current experimental data. To reduce such a wide possibility is an issue of great importance to our purpose to find the minimal form of mass matrices, which might reveal a deeper understanding of the structure of Yukawa couplings. A possible physical difference among the matrix patterns would appear in yet unknown phenomena involving right-handed neutrinos. In the following, we discuss flavor-violating decay of charged leptons and the baryon asymmetry of the universe via thermal leptogenesis for typical examples of mass matrices in order to clarify some physical differences among the viable seesaw mass textures.

### 6.1 Flavor-violating decay of charged leptons

Flavor-changing couplings in the lepton Yukawa sector generate the observable sign of neutrino flavor oscillations. Other physical effects are possible in physics beyond the standard model and helpful to discriminate the patterns of Yukawa matrices. A well-known example we will discuss below is that, if the theory is supersymmetrized, flavor violation in Yukawa couplings is translated to off-diagonal components of scalar masses through radiative corrections. That could induce sizable rates of flavor-changing rare decays for charged leptons [28] even with the flavor-universal condition for supersymmetry-breaking couplings motivated by the minimal supergravity theory.

There are three types of supersymmetry-breaking parameters relevant to the analysis: the mass parameters of gauge fermions  $M_{\lambda_a}$  ( $a = 1, 2, 3$ ), the mass and trilinear couplings of scalar partners of quarks and leptons,  $(m_x^2)_{ij}$  and  $(a_y)_{ij}$  ( $i, j = 1, 2, 3$ ). The latter two couplings have generation indices  $i, j$  and would be additional sources of flavor violation. In the following, we take, as a conservative assumption, the flavor-universal boundary conditions at some high-energy scale  $\Lambda$ :  $(m_x^2)_{ij} = m_0^2 \delta_{ij}$  and  $(a_y)_{ij} = a_0 \delta_{ij}$ . However the flavor universality is disturbed at quantum level by loop corrections involving scalar quarks and leptons. For example, the renormalization-group running from  $\Lambda$  down to the right-handed neutrino mass scale induces off-diagonal elements of left-handed scalar lepton masses:

$$(m_l^2)_{ij} \sim \frac{1}{8\pi^2} (3m_0^2 + a_0^2) \sum_k (\bar{Y}_\nu^\dagger)_{ik} (\bar{Y}_\nu)_{kj} \ln \left( \frac{\Lambda}{M_{R_k}} \right), \quad (i \neq j). \quad (6.1)$$

The neutrino Yukawa matrix  $\bar{Y}_\nu$  is evaluated in the generation basis where the charged lepton Yukawa matrix  $Y_e$  and right-handed Majorana mass matrix  $M_R$  are flavor diagonal [ $M_{R_i}$  are the (positive) eigenvalues of  $M_R$ ]. With non-vanishing off-diagonal elements at hand, flavor-violating processes such as  $\mu \rightarrow e\gamma$  and  $\mu \rightarrow e$  conversion in nuclei generally occurs through the mediation of scalar leptons in loop diagrams. The branching ratio for the  $\mu \rightarrow e\gamma$  decay is usually dominated by the gaugino-higgsino mixing diagram, and is roughly estimated as

$$\text{Br}(\mu \rightarrow e\gamma) \simeq \frac{3\alpha}{2\pi} \cdot \frac{m_W^4 |(m_l^2)_{12}|^2}{M_{\text{susy}}^8} \tan^2 \beta, \quad (6.2)$$

where  $\alpha$  is the fine structure constant, and  $m_W$  and  $M_{\text{susy}}$  denote the mass scales of the  $W$  boson and typical superparticles in the loops, respectively. The decay rate is enhanced by  $\tan \beta$ , the ratio of vacuum expectation values of two Higgs doublets in minimal supersymmetric models. In the numerical evaluation below, we will also include all other contributions to the  $\mu \rightarrow e\gamma$  decay amplitude. The flavor-violating decay of the muon might be therefore observed if sizable flavor-changing supersymmetry-breaking couplings are generated through radiative corrections which depend on the structure of lepton Yukawa matrices.

For illustrative examples, we take two types of minimal mass textures, which have been found in the previous section to be consistent with the current experimental data, and study their predictions for charged lepton rare decay, in particular, for  $\mu \rightarrow e\gamma$ . The first example is a combination of textures (see table 3 in the appendix),

$$Y_\nu = \begin{pmatrix} p & q \\ & r \\ & & s \end{pmatrix}, \quad M_R = \begin{pmatrix} & t \\ t & \\ & & u \end{pmatrix}. \quad (6.3)$$

All the parameters can be made real and positive by phase rotations of neutrino fields. After integrating out heavy right-handed neutrinos, one obtains effective Majorana mass matrix for light neutrinos:

$$M_L = - \begin{pmatrix} pr/t & & \\ pr/t & 2qr/t & \\ & & s^2/u \end{pmatrix} \bar{v}^2, \quad (6.4)$$

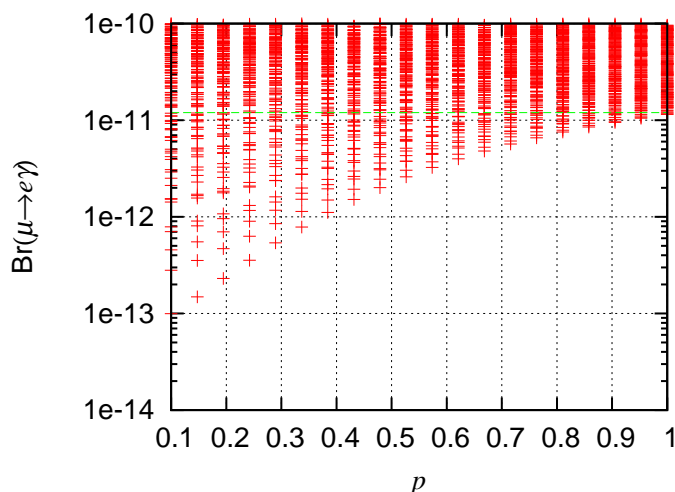
where  $\bar{v}$  is a vacuum expectation value of the Higgs field which gives neutrino Dirac masses. This matrix  $M_L$  takes one of the minimal forms found in section 4.2 [the matrix (4.8)]. According to the physical predictions presented there, we find that several (combinations of) parameters are fixed by the observables:

$$\frac{p}{q} \simeq \tan 2\theta_{12}, \quad \frac{2r\bar{v}^2}{t} \sqrt{pq} \simeq \sqrt{\Delta m_{21}^2 \sin 2\theta_{12}}, \quad \frac{(s\bar{v})^2}{u} \simeq \sqrt{\Delta m_{32}^2}, \quad (6.5)$$

neglecting  $\mathcal{O}(10^{-2})$  corrections. The atmospheric neutrino oscillation is explained by the charged lepton sector, and leads to no significant restriction on the neutrino side. With the textures (6.3), a pair of off-diagonal elements of scalar lepton mass matrix is radiatively generated through renormalization-group running, which is found to be proportional to

$$(\bar{Y}_\nu^\dagger \bar{Y}_\nu)_{21} \simeq \frac{1}{\sqrt{2}} \left[ pq + (-p^2 + q^2 + r^2)(V_{eL})_{21} + (p^2 - s^2)(V_{eL})_{31} \right]. \quad (6.6)$$

Notice that, since the first-two generation right-handed neutrinos are degenerate for  $M_R$  (6.3), the basis rotation which diagonalizes  $M_R$  is canceled out in the expression (6.6). In the limit  $p = r = s$ , the experimental relations (6.5) are still satisfied with appropriate values of other parameters, while the off-diagonal element (6.6) does not vanish and gives rise to a unsuppressed decay rate of  $\mu \rightarrow e\gamma$  proportional to  $q^2$ .



**Figure 4:** A plot of the branching ratio of  $\mu \rightarrow e\gamma$  for the texture example (6.3). The horizontal axis denotes a Yukawa coupling constant. The horizontal (green) line shows the current experimental upper bound of the branching ratio. The universal scalar mass parameter and gaugino masses are taken to be  $m_0 = M_{\lambda_1} = M_{\lambda_2} = M_{\lambda_3} = 800$  GeV and  $a_0 = 0$  at the unification scale. The branching ratio is scaled with  $\tan^2 \beta$ , and here we assume  $\tan \beta = 10$ .

Figure 4 shows a numerical result of the  $\mu \rightarrow e\gamma$  decay rate for the texture example (6.3). In this figure, the neutrino Yukawa couplings  $p$ ,  $r$ , and  $s$  vary between 0.1 and 1, while the other parameters  $q$ ,  $t$ , and  $u$  are fixed so that the neutrino oscillation data is reproduced. We assume, as an example, the universal mass parameter  $m_0$  and gaugino masses  $M_{\lambda_{1,2,3}}$  to be 800 GeV at the unification scale, and  $\tan \beta = 10$ . It can be seen from the figure that, for larger values of Yukawa couplings, the flavor structure (6.3) is already excluded by the absence of experimental signature of charged lepton rare decay.

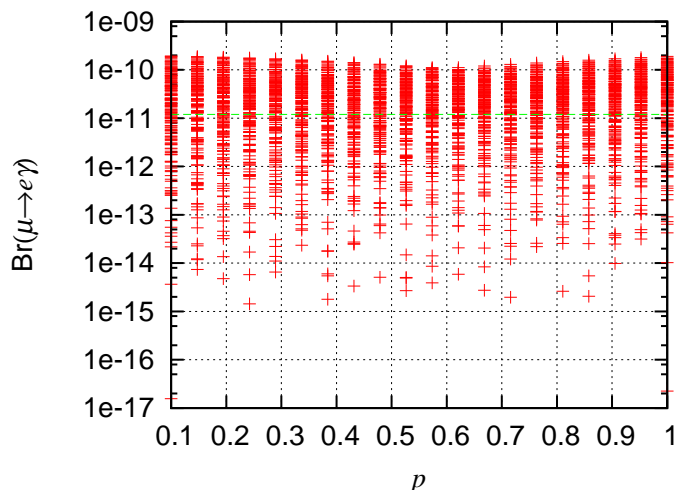
Another texture example we consider is a combination (see table 4 in the appendix),

$$Y_\nu = \begin{pmatrix} p \\ q \\ r \end{pmatrix}, \quad M_R = \begin{pmatrix} s & \\ s & t \\ & u \end{pmatrix}. \quad (6.7)$$

All the matrix elements can be made real and positive. It may be interesting to note that, among the minimal forms of neutrino mass matrices presented in the appendix, only this texture example leads to the same form of  $M_L$  as the previous one (6.3). This is because the minimal mass matrices (6.7) are chosen for the second example as a comparison. In the present case, the induced Majorana mass matrix for light neutrinos is

$$M_L = - \begin{pmatrix} pq/s & & \\ pq/s & -p^2 t/s^2 & \\ & & r^2/u \end{pmatrix} \bar{\nu}^2, \quad (6.8)$$

which is of the identical form to the first example and gives the same predictions for neutrino masses and mixing angles. However the flavor structure of neutrino Yukawa



**Figure 5:** A plot of the branching ratio of  $\mu \rightarrow e\gamma$  for the texture example (6.7). The horizontal axis denotes a Yukawa coupling constant. The horizontal (green) line shows the current experimental upper bound of the branching ratio. The universal scalar mass parameters and gaugino masses are taken at the unification scale to be the same as in figure 4. Here we assume  $\tan\beta = 10$ .

matrix is different from (6.6):

$$(\bar{Y}_\nu^\dagger \bar{Y}_\nu)_{21} \simeq \frac{1}{\sqrt{2}} \left[ (p^2 - q^2)(V_{eL})_{21} + (q^2 - r^2)(V_{eL})_{31} \right], \quad (6.9)$$

which contains 3 free parameters  $p$ ,  $q$ , and  $r$ . The other matrix elements are determined by the physical observables similarly to (6.5). The off-diagonal element (6.9) describes induced flavor-violating masses of scalar leptons approximately in the situation that there is no large mass hierarchy among the right-handed neutrinos. Note, in the present case, that one can take the limit  $p = q = r$  without being incompatible with the experimental data, which limit suppresses the amplitude of  $\mu \rightarrow e\gamma$  up to tiny contribution from right-handed scalar lepton masses. Thus the textures (6.7) generally tend to predict smaller branching ratio of charged lepton rare decay even with large neutrino Yukawa couplings of  $\mathcal{O}(1)$ .

In figure 5, we present a numerical result of the  $\mu \rightarrow e\gamma$  decay rate for the texture example (6.7). In this figure, the neutrino Yukawa couplings  $p$ ,  $q$ , and  $r$  vary between 0.1 and 1. The supersymmetry-breaking parameters are taken to be the same as in figure 4. The figure shows that the branching ratio can be suppressed ( $< 10^{-12}$ ) even for a relatively large Yukawa coupling ( $0.4 < p < 1$ ), which fact makes a sharp contrast with the first texture example (see figure 4).

In this way, the measurement of flavor-changing rare decay of charged leptons would be able to split the degeneracy of possible texture forms of neutrinos.

## 6.2 Thermal leptogenesis

Another interesting physical effect of neutrino mass textures comes from the existence of high-energy CP violation in the Yukawa couplings. That is, in some classes of texture

forms, there remain complex phases of couplings which cannot be rotated away by the redefinition of neutrino fields. Such phase degrees of freedom may explain the baryon asymmetry of the universe through the thermal leptogenesis [29].

Let us consider the following texture form, which is one of the minimal neutrino mass matrices found in section 5 (see table 3 in the appendix):

$$Y_\nu = \begin{pmatrix} pe^{i\phi} & & \\ q & r & \\ & & s \end{pmatrix}, \quad M_R = \begin{pmatrix} & t & \\ t & & \\ & & u \end{pmatrix}. \quad (6.10)$$

To study CP-violating effects, it is convenient to reduce the phase degrees of freedom in the Yukawa couplings. In the above matrix forms, we have taken a basis where all the parameters but  $(Y_\nu)_{11}$  are real (and positive) by redefining neutrino fields. The resultant Majorana mass matrix for light neutrinos becomes

$$M_L = - \begin{pmatrix} 2pqe^{2i\phi}/t & pr/t \\ pr/t & s^2/u \end{pmatrix} \bar{\nu}^2. \quad (6.11)$$

This form of  $M_L$  predicts the inverted mass hierarchy of neutrinos, detectable neutrino-less double beta decay, and low-energy CP-violation in future neutrino oscillation experiments which is found to be proportional to  $\sin(\phi + \phi_e) \cos \phi$ , where  $\phi_e$  originates from unremovable phases of charged lepton Yukawa couplings.

The CP asymmetry generated in the decay of the  $i$ -th generation right-handed neutrino is described by the quantity

$$\epsilon_i = \frac{\sum_j \Gamma(\nu_{R_i} \rightarrow L_j H) - \sum_j \Gamma(\nu_{R_i} \rightarrow L_j^c H^\dagger)}{\sum_j \Gamma(\nu_{R_i} \rightarrow L_j H) + \sum_j \Gamma(\nu_{R_i} \rightarrow L_j^c H^\dagger)}, \quad (6.12)$$

and the resultant leptonic asymmetry  $\eta_L$  [30] is evaluated as

$$\eta_L \simeq \frac{1}{g_*} \sum_i \kappa_i \epsilon_i, \quad (6.13)$$

where  $\kappa_i$  are the factors which represent wash-out effects governed by the Boltzmann equations, and  $g_*$  is the number of relativistic degrees of freedom. An approximate analytic expression for the factor  $\kappa_i$  is given by

$$\kappa_i \simeq 0.3 \left( \frac{10^{-3} \text{eV}}{\tilde{m}_i} \right) \left( \ln \frac{\tilde{m}_i}{10^{-3} \text{eV}} \right)^{-0.6}, \quad \tilde{m}_i \equiv (M_\nu M_\nu^\dagger)_{ii} / M_{R_i}, \quad (6.14)$$

if right-handed neutrinos are enough strongly coupled to the thermal bath:  $\tilde{m}_i > 10^{-3}$  eV. The electroweak anomaly which is in thermal equilibrium at the temperature  $10^2 \sim 10^{12}$  GeV converts a part of  $B - L$  asymmetry into  $B$  asymmetry in such a way that  $\eta_B = -28/51 \times \eta_L$  in the standard model [31].

In addition to the existence of CP-violating phase, the matrix forms (6.10) are also interesting in that the right-handed neutrino masses are degenerate in the first and second



generations. Such a degeneracy is known to enhance the leptonic asymmetry with self-energy contribution in the decay of right-handed neutrinos [32]. That helps to achieve the observed range of the baryon asymmetry with relatively light right-handed neutrinos, which is reasonable from a viewpoint of the cosmological problem of unstable gravitino in supersymmetric theory [33]. We discuss below the possibility of such enhancement of lepton asymmetry for the textures (6.10).

In the region where  $\nu_{R_1}$  and  $\nu_{R_2}$  are nearly degenerate in mass, the CP-asymmetry parameters are well approximated as [32]

$$\epsilon_1 \simeq \frac{\text{Im } A_{12}^2}{A_{11}} \cdot \frac{R}{R^2 + A_{22}^2}, \quad \epsilon_2 \simeq \frac{\text{Im } A_{12}^2}{A_{22}} \cdot \frac{R}{R^2 + A_{11}^2}, \quad (6.15)$$

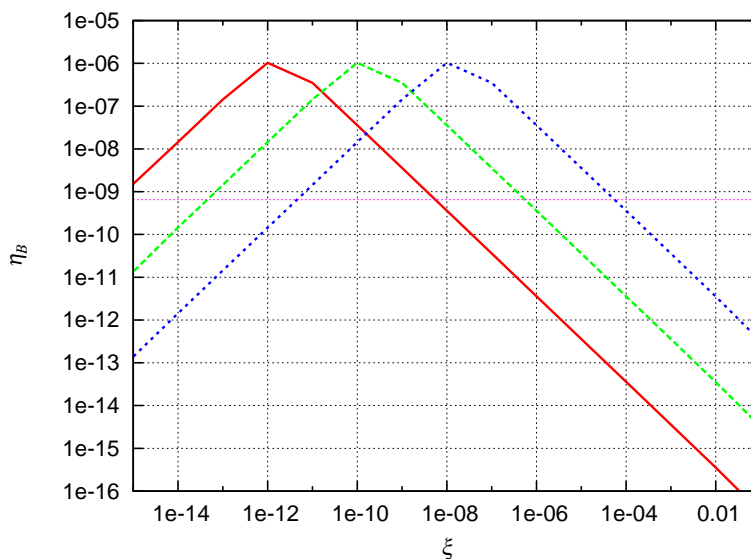
where  $A_{ij} = (Y_\nu Y_\nu^\dagger)_{ij}/8\pi$  and  $R = (M_{R_1}^2 - M_{R_2}^2)/M_{R_1}M_{R_2}$ . The mass eigenvalues of right-handed neutrinos  $M_{R_i}$  are taken to be real and positive. It is noticed that there exist the regulating terms,  $A_{11}^2$  and  $A_{22}^2$ , which come from the resummation of self-energy diagrams.

If the degeneracy of two heavy Majorana neutrinos is exact ( $R \rightarrow 0$ ), the asymmetry parameters (6.15) vanish. In the present case we are discussing, the matrix  $M_R$  (6.10) means the equal mass eigenvalues for the first-two right-handed neutrinos. However such an exact degeneracy may be relaxed, for example, by radiative corrections from other sectors such as  $Y_\nu$ . In fact, a required order of mass splitting is generally small and could easily be realized. We define the parameter  $\xi$  to describe the degree of degeneracy for  $M_{R_1}$  and  $M_{R_2}$ :

$$\frac{M_{R_2}}{M_{R_1}} = 1 - \xi. \quad (6.16)$$

Using (6.14) and (6.15), we can calculate the baryon asymmetry. The predicted baryon asymmetry as the function of  $\xi$  is shown in figure 6. The red-solid, green-dashed, and blue-dotted lines show the predictions for  $M_{R_1} \simeq M_{R_2} = 10^5, 10^7, \text{ and } 10^9$  GeV, respectively. The other masses and Yukawa coupling constants are determined so that the results from neutrino oscillation experiments are satisfied. As seen from the figure, the observed baryon asymmetry of the universe  $\eta_B = (6.2 - 6.9) \times 10^{-10}$  [34] is reproduced for each parameter set with a small mass splitting  $\xi$ . Recent studies on the decay of gravitino suggest that the reheating temperature of the universe is preferred to be less than  $10^7$  GeV for successful big-bang nucleosynthesis with the unstable gravitino lighter than 10 TeV [33]. We here found that the structure of mass matrices (6.10) is capable of explaining the observed baryon asymmetry while solving the cosmological gravitino problem. It would be an interesting task to analyze the other minimal forms of neutrino mass matrices presented in the appendix whether they can reproduce the proper value of  $\eta_B$  in the leptogenesis scenario.

Finally we comment on the leptonic CP violation. In figure 6, we have set the unremovable complex phase in the Yukawa couplings as  $\phi = \pi/2$ . This does not lead to low-energy CP violation in future neutrino oscillation experiments, which is proportional to  $\cos \phi$ . However that is just an illustrative example, and the detailed analysis of CP-violating effects discriminating possible forms of minimal lepton mass matrices is left to future investigation.



**Figure 6:** The baryon-to-photon ratio as the function of the degeneracy parameter  $\xi$  for one of the minimal forms of seesaw mass matrices((6.10)). The horizontal dotted line shows the observed value of baryon asymmetry. The red-solid, green-dashed, and blue-dotted lines show the predictions for  $(p, M_{R_1}) = (10^{-5}, 10^5 \text{ GeV})$ ,  $(10^{-4}, 10^7 \text{ GeV})$ , and  $(10^{-3}, 10^9 \text{ GeV})$ , respectively. For other parameters, we take  $M_{R_2} = (1 - \xi)M_{R_1}$ ,  $M_{R_3} = 10^2 M_{R_1}$ ,  $m_1 = 4.61 \times 10^{-2} \text{ eV}$ ,  $m_2 = 4.69 \times 10^{-2} \text{ eV}$ ,  $\theta_{12} = 33^\circ$ , and  $\phi = \pi/2$ .

## 7. Summary

In this paper, we have studied mass matrices of quarks and leptons in unified theory which have as many vanishing elements as possible. First we have found that the following form of mass matrices is minimal and consistent with the current experimental data of quarks and charged leptons, including the recent measurement of CP violation in the B-meson system:

$$M_u = \begin{pmatrix} a & & \\ a & b & \\ & b & c \end{pmatrix}, \quad M_d = \begin{pmatrix} e & & \\ d & h & \\ & g & f \end{pmatrix}, \quad M_e = \begin{pmatrix} e & & \\ d & -3h & \\ & g & f \end{pmatrix}. \quad (7.1)$$

Trivial exchanges of generation labels of  $Q_i$  and  $u_{R_i}$  do not change physical predictions and lead to viable solutions. An interesting observation in the matrix form (7.1) is that it induces a large mixture of left-handed charged leptons between the second and third generations, which is suitable to the experimental results for the atmospheric neutrinos.

Bases on this matrix form of charged fermions, we have investigated two types of mass matrices for neutrinos. The first analysis deals with the effective Majorana mass matrix  $M_L$  of left-handed neutrinos. We have found that the following forms are minimal, i.e. contain the maximal number of vanishing elements:

$$M_L = \begin{pmatrix} l & & \\ l & m & \\ & & n \end{pmatrix}, \begin{pmatrix} l & & \\ l & m & \\ & m & n \end{pmatrix}, \begin{pmatrix} l & m & \\ & m & n \end{pmatrix}, \begin{pmatrix} l & & \\ & m & \\ l & m & n \end{pmatrix}. \quad (7.2)$$

Combined with (7.1), all current experimental results of quarks and leptons are explained. Trivial exchanges of generation labels of  $L_i$  (and  $d_{R_i}$  in unified theory) in (7.1) and (7.2) do not change physical predictions and lead to viable solutions. In addition, the exchange of the second and third labels in  $M_L$  also gives the matrices experimentally allowed, because of a large mixture of left-handed charged leptons between the second and third generations mentioned above. Each set of minimal mass textures predicts typical mass spectrum and generation mixing of neutrinos (table 2), and is therefore testable with precise measurements of masses and mixing angles in future neutrino experiments.

The second type of our analysis introduces heavy right-handed neutrinos, and discuss the minimality of neutrino Dirac mass matrix  $M_\nu$  and Majorana mass matrix  $M_R$  of right-handed neutrinos. In this case, we have found a variety of successful mass textures presented in the appendix, which are classified into three categories:

- $M_\nu$  with 5 zeros +  $M_R$  with symmetric 4 zeros (12 patterns)
- $M_\nu$  with 6 zeros +  $M_R$  with symmetric 3 zeros (25 patterns)
- $M_\nu$  with 7 zeros +  $M_R$  with symmetric 2 zeros (6 patterns)

Integrated out heavy right-handed neutrinos, almost all the patterns effectively induce  $M_L$  in the forms of (7.2), but the other 5 patterns generate different forms of  $M_L$ .

As typical phenomenological implications of the minimal mass matrices of neutrinos, we have discussed the  $\mu \rightarrow e\gamma$  decay and the thermal leptogenesis scenario. In the study of  $\mu \rightarrow e\gamma$ , some of textures are found to predict large decay amplitudes in natural parameter regions to reach the experimental upper bound. It also turns out that mass matrices with unremovable complex phases of matrix elements can explain the observed baryon asymmetry of the universe via thermal leptogenesis. In particular, some types of textures avoid an apparent inconsistency between viable thermal leptogenesis and the gravitino problem in supersymmetric theory by virtue of resonant behavior in the decays of right-handed neutrinos. Future neutrino experiments for low-energy leptonic CP violation and neutrino-less double beta decay would be expected to distinguish the minimal forms of quark and lepton mass matrices.

## Acknowledgments

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## A. The minimal forms of seesaw mass textures

In this appendix, we present the minimal forms of neutrino mass matrices in the seesaw scenario (see section 5). These matrices are consistent with the current experimental data of quarks and leptons (2.7)–(2.10) and (2.12) in collaboration with the minimal solution (3.9) for charged fermions.

The following tables show experimentally viable forms of  $M_\nu$  and  $M_R$ . It should be noted that, in addition to the solutions presented in the tables, there are two types of additional matrix forms which are obtained by exchanging generation indices (rows and/or columns). The first is the label exchange of right-handed neutrinos. In the seesaw formula, the simultaneous exchange of the same generation indices of  $\nu_{R_i}$  in  $M_\nu$  and  $M_R$  does not affect physical consequences at low-energy regime. Secondly, since the charged lepton mixing matrix  $V_{eL}$  is approximately invariant under the exchange of the second and third rows ( $\nu_{L_2} \leftrightarrow \nu_{L_3}$ ), the same exchange in  $M_\nu$  also gives available matrix forms.

In the tables, we show several results for each combination of neutrino mass matrices: “ $M_L$ ”, “spectrum”, “CP”, and “ $0\nu\beta\beta$ ”. The column  $M_L$  shows the mass matrix forms of left-handed neutrinos effectively induced after the seesaw operation. In the column “spectrum”, we present possible mass patterns of light neutrinos for each texture. The abbreviation NH denotes the normal mass hierarchy  $m_1 < m_2 \ll m_3$  and IH the inverted one  $m_3 \ll m_1 < m_2$ . The column “CP” shows that matrices contain unremovable complex phases, which may be required to implement the leptogenesis and also leads to low-energy CP violation in neutrino oscillation experiments. The symbol  $\bigcirc$  means that there exist phase degrees of freedom in  $M_\nu$  and  $M_R$  which cannot be rotated away by the redefinition of neutrino fields, and  $\times$  means the absence of such complex phases. Finally, for “ $0\nu\beta\beta$ ”, the symbol  $\bigcirc$  ( $\times$ ) implies that the averaged mass  $|\langle m_{ee} \rangle|$  is larger (smaller) than  $10^{-2}$  eV. The averaged neutrino mass for  $0\nu\beta\beta$  is defined for the parametrization of the MNS matrix given in section 2 as  $\langle m_{ee} \rangle = m_1 \cos^2 \theta_{12} \cos^2 \theta_{13} e^{-2i\rho} + m_2 \sin^2 \theta_{12} \cos^2 \theta_{13} e^{-2i\sigma} + m_3 \sin^2 \theta_{13} e^{2i\delta}$ .

$M_\nu$	$M_R$	$M_L$	spectrum	CP	$0\nu\beta\beta$
$\begin{pmatrix} p & q \\ r & s \end{pmatrix}$	$\begin{pmatrix} t \\ t & u \\ u \end{pmatrix}$	$\rightarrow \begin{pmatrix} l \\ l & m \\ m & n \end{pmatrix}$	NH	$\times$	$\times$
$\begin{pmatrix} p \\ q \\ r & s \end{pmatrix}$	$\begin{pmatrix} t \\ t & u \\ u \end{pmatrix}$	$\rightarrow \begin{pmatrix} l \\ l & m \\ m & n \end{pmatrix}$	NH	$\times$	$\times$
$\begin{pmatrix} p \\ q & r \\ s \end{pmatrix}$	$\begin{pmatrix} t \\ t & u \\ u \end{pmatrix}$	$\rightarrow \begin{pmatrix} l & m \\ m & n \end{pmatrix}$	IH	$\circ$	$\circ$
$\begin{pmatrix} p \\ q & r \\ s \end{pmatrix}$	$\begin{pmatrix} t \\ t & u \\ u \end{pmatrix}$	$\rightarrow \begin{pmatrix} l & m \\ m & n \end{pmatrix}$	IH	$\circ$	$\circ$
$\begin{pmatrix} p \\ q & r \\ s \end{pmatrix}$	$\begin{pmatrix} t \\ u \\ t \end{pmatrix}$	$\rightarrow \begin{pmatrix} l & m \\ m & n \end{pmatrix}$	IH	$\circ$	$\circ$
$\begin{pmatrix} p & q \\ r \\ s \end{pmatrix}$	$\begin{pmatrix} t \\ t & u \\ u \end{pmatrix}$	$\rightarrow \begin{pmatrix} l & m \\ m & n \end{pmatrix}$	IH	$\circ$	$\circ$
$\begin{pmatrix} p & q \\ r \\ s \end{pmatrix}$	$\begin{pmatrix} t \\ t & u \\ u \end{pmatrix}$	$\rightarrow \begin{pmatrix} l & m \\ m & n \end{pmatrix}$	IH	$\circ$	$\circ$
$\begin{pmatrix} p \\ q & r & s \end{pmatrix}$	$\begin{pmatrix} t \\ u \\ u \end{pmatrix}$	$\rightarrow \begin{pmatrix} l \\ m \\ l & m & n \end{pmatrix}$	NH	$\times$	$\times$
$\begin{pmatrix} p & q \\ r & s \end{pmatrix}$	$\begin{pmatrix} t \\ u \\ u \end{pmatrix}$	$\rightarrow \begin{pmatrix} l & m \\ l & n \\ m & n \frac{2mn}{t} \end{pmatrix}$	NH	$\times$	$\times$
$\begin{pmatrix} p & q \\ r & s \end{pmatrix}$	$\begin{pmatrix} t \\ u \\ u \end{pmatrix}$	$\rightarrow \begin{pmatrix} l & m \\ m & n \end{pmatrix}$	IH	$\times$	$\circ$
$\begin{pmatrix} p & q \\ r \\ s \end{pmatrix}$	$\begin{pmatrix} t \\ u \\ u \end{pmatrix}$	$\rightarrow \begin{pmatrix} \frac{l^2}{m} & l \\ l & m \\ n \end{pmatrix}$	NH	$\times$	$\times$
$\begin{pmatrix} p & q \\ r \\ s \end{pmatrix}$	$\begin{pmatrix} t \\ u \\ t \end{pmatrix}$	$\rightarrow \begin{pmatrix} l \\ m \\ l & m & n \end{pmatrix}$	NH	$\times$	$\times$

**Table 3:**  $M_\nu$  with 5 zeros +  $M_R$  with symmetric 4 zeros.

$M_\nu$	$M_R$	$M_L$	spectrum	CP	$0\nu\beta\beta$
$\begin{pmatrix} p \\ q \\ r \end{pmatrix}$	$\begin{pmatrix} s \\ s t \\ u \end{pmatrix}$	$\rightarrow \begin{pmatrix} l \\ l m \\ n \end{pmatrix}$	NH	×	×
$\begin{pmatrix} p \\ q \\ r \end{pmatrix}$	$\begin{pmatrix} s \\ s t \\ u \end{pmatrix}$	$\rightarrow \begin{pmatrix} l m \\ m n \end{pmatrix}$	IH	×	○
$\begin{pmatrix} p \\ q \\ r \end{pmatrix}$	$\begin{pmatrix} s \\ s t \\ u \end{pmatrix}$	$\rightarrow \begin{pmatrix} l m \\ m n \end{pmatrix}$	IH	×	○
$\begin{pmatrix} p \\ q \\ r \end{pmatrix}$	$\begin{pmatrix} s & t \\ s & t \\ t & u \end{pmatrix}$	$\rightarrow \begin{pmatrix} l m \\ m n \end{pmatrix}$	IH	×	○
$\begin{pmatrix} p \\ q \\ r \end{pmatrix}$	$\begin{pmatrix} s & t \\ s & t \\ t & u \end{pmatrix}$	$\rightarrow \begin{pmatrix} l m \\ m n \end{pmatrix}$	IH	×	○
$\begin{pmatrix} p \\ q \\ r \end{pmatrix}$	$\begin{pmatrix} s & t \\ s & t \\ t & u \end{pmatrix}$	$\rightarrow \begin{pmatrix} l m \\ m n \end{pmatrix}$	IH	×	○
$\begin{pmatrix} p \\ q \\ r \end{pmatrix}$	$\begin{pmatrix} s & t \\ s & t \\ t & u \end{pmatrix}$	$\rightarrow \begin{pmatrix} l m \\ m n \end{pmatrix}$	IH	×	○
$\begin{pmatrix} p q \\ r \end{pmatrix}$	$\begin{pmatrix} s \\ t \\ u \end{pmatrix}$	$\rightarrow \begin{pmatrix} \frac{l^2}{m} l \\ l m \\ n \end{pmatrix}$	NH	×	×
$\begin{pmatrix} p q \\ r \end{pmatrix}$	$\begin{pmatrix} s \\ t u \\ u \end{pmatrix}$	$\rightarrow \begin{pmatrix} l \\ m \\ l m n \end{pmatrix}$	NH	×	×
$\begin{pmatrix} p q \\ r \end{pmatrix}$	$\begin{pmatrix} s t \\ t u \\ u \end{pmatrix}$	$\rightarrow \begin{pmatrix} l \\ m \\ l m n \end{pmatrix}$	NH	×	×
$\begin{pmatrix} p q \\ r \end{pmatrix}$	$\begin{pmatrix} s \\ s t \\ u \end{pmatrix}$	$\rightarrow \begin{pmatrix} \frac{l^2}{m} l \\ l m \\ n \end{pmatrix}$	NH	×	×
$\begin{pmatrix} p q \\ r \end{pmatrix}$	$\begin{pmatrix} s \\ t u \\ u \end{pmatrix}$	$\rightarrow \begin{pmatrix} \frac{l^2}{m} l \\ l m \\ n \end{pmatrix}$	NH	×	×

**Table 4:**  $M_\nu$  with 6 zeros +  $M_R$  with symmetric 3 zeros.

$M_\nu$	$M_R$	$M_L$	spectrum	CP	$0\nu\beta\beta$
$\begin{pmatrix} p & q \\ r \end{pmatrix}$	$\begin{pmatrix} s \\ t \\ u \end{pmatrix}$	$\rightarrow \begin{pmatrix} l & m \\ m & n \end{pmatrix}$	IH	$\circ$	$\circ$
$\begin{pmatrix} p & q \\ r \end{pmatrix}$	$\begin{pmatrix} s & t \\ s & t \\ t & u \end{pmatrix}$	$\rightarrow \begin{pmatrix} l & m \\ m & n \end{pmatrix}$	IH	$\circ$	$\circ$
$\begin{pmatrix} p & q \\ r \end{pmatrix}$	$\begin{pmatrix} s & t \\ s & t \\ t & u \end{pmatrix}$	$\rightarrow \begin{pmatrix} l & m \\ m & n \end{pmatrix}$	IH	$\circ$	$\circ$
$\begin{pmatrix} p \\ q & r \end{pmatrix}$	$\begin{pmatrix} s \\ s & t \\ t & u \end{pmatrix}$	$\rightarrow \begin{pmatrix} l & m \\ m & n \end{pmatrix}$	IH	$\circ$	$\circ$
$\begin{pmatrix} p \\ q & r \end{pmatrix}$	$\begin{pmatrix} s \\ s & t \\ t & u \end{pmatrix}$	$\rightarrow \begin{pmatrix} l & m \\ m & n \end{pmatrix}$	IH	$\circ$	$\circ$
$\begin{pmatrix} p & q \\ r \end{pmatrix}$	$\begin{pmatrix} s & t \\ s & t \\ t & u \end{pmatrix}$	$\rightarrow \begin{pmatrix} l & m \\ m & n \end{pmatrix}$	IH	$\circ$	$\circ$
$\begin{pmatrix} p & q \\ r \end{pmatrix}$	$\begin{pmatrix} s & t \\ s & t \\ t & u \end{pmatrix}$	$\rightarrow \begin{pmatrix} l & m \\ m & n \end{pmatrix}$	IH	$\circ$	$\circ$
$\begin{pmatrix} p & q \\ r \end{pmatrix}$	$\begin{pmatrix} s & t \\ s & t \\ u \end{pmatrix}$	$\rightarrow \begin{pmatrix} l & m \\ m & n \end{pmatrix}$	IH	$\circ$	$\circ$
$\begin{pmatrix} p & q \\ r \end{pmatrix}$	$\begin{pmatrix} s & t \\ s & t \\ u \end{pmatrix}$	$\rightarrow \begin{pmatrix} l & m \\ m & n \end{pmatrix}$	IH	$\circ$	$\circ$
$\begin{pmatrix} p \\ q & r \end{pmatrix}$	$\begin{pmatrix} s & t \\ s & t \\ u \end{pmatrix}$	$\rightarrow \begin{pmatrix} l & m \\ m & n \end{pmatrix}$	IH	$\circ$	$\circ$
$\begin{pmatrix} p \\ q & r \end{pmatrix}$	$\begin{pmatrix} s & t \\ s & t \\ u \end{pmatrix}$	$\rightarrow \begin{pmatrix} l & m \\ m & n \end{pmatrix}$	IH	$\circ$	$\circ$
$\begin{pmatrix} p & q \\ r \end{pmatrix}$	$\begin{pmatrix} s & t \\ s & t \\ u \end{pmatrix}$	$\rightarrow \begin{pmatrix} l & m \\ m & n \end{pmatrix}$	IH	$\circ$	$\circ$
$\begin{pmatrix} p & q \\ r \end{pmatrix}$	$\begin{pmatrix} s & t \\ s & t \\ u \end{pmatrix}$	$\rightarrow \begin{pmatrix} l & m \\ m & n \end{pmatrix}$	IH	$\circ$	$\circ$

Table 4: (continued.)

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$M_\nu$	$M_R$	$M_L$	spectrum	CP	$0\nu\beta\beta$
$\begin{pmatrix} p \\ q \end{pmatrix}$	$\begin{pmatrix} r & s \\ s & t \\ & u \end{pmatrix}$	$\rightarrow \begin{pmatrix} l & m \\ m & n \end{pmatrix}$	IH	$\bigcirc$	$\bigcirc$
$\begin{pmatrix} p \\ q \end{pmatrix}$	$\begin{pmatrix} r \\ s & t \\ r & t & u \end{pmatrix}$	$\rightarrow \begin{pmatrix} l & m \\ m & n \end{pmatrix}$	IH	$\bigcirc$	$\bigcirc$
$\begin{pmatrix} p \\ q \end{pmatrix}$	$\begin{pmatrix} r \\ s & t \\ r & t & u \end{pmatrix}$	$\rightarrow \begin{pmatrix} l & m \\ m & n \end{pmatrix}$	IH	$\bigcirc$	$\bigcirc$
$\begin{pmatrix} p \\ q \end{pmatrix}$	$\begin{pmatrix} r & s \\ r & t \\ s & u \end{pmatrix}$	$\rightarrow \begin{pmatrix} l & m \\ m & n \end{pmatrix}$	IH	$\bigcirc$	$\bigcirc$
$\begin{pmatrix} p \\ q \end{pmatrix}$	$\begin{pmatrix} r & s \\ r & t \\ s & u \end{pmatrix}$	$\rightarrow \begin{pmatrix} l & m \\ m & n \end{pmatrix}$	IH	$\bigcirc$	$\bigcirc$
$\begin{pmatrix} p \\ q \end{pmatrix}$	$\begin{pmatrix} r & s \\ r & t \\ s & t & u \end{pmatrix}$	$\rightarrow \begin{pmatrix} l & m \\ m & n \end{pmatrix}$	IH	$\bigcirc$	$\bigcirc$

**Table 5:**  $M_\nu$  with 7 zeros +  $M_R$  with symmetric 2 zeros.

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